Calculation of the reflected wave from a pipe with a nozzle end by the Lax–Wendroff method

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The solution of gas flow problems in pipes with nozzle ends is discussed. The Lax–Wendroff method, with a hybrid boundary condition approximation, is used to compute the numerical solutions to some test problems, The accuracy of the solutions obtained by this method is assessed by a comparison with theoretical solutions.

Keywords: *gasflow, Lax–Wendroff method, boundary conditions*

Calculation of the reflected wave arising from a shock impinging on the closed, open or nozzle end of a pipe by graphical methods is well-established[1]. Another possibility is to solve the problem on a computer using either a 'shock-tracking' or 'shock-capturing' approach.

In a shock-tracking method, the relations across any discontinuity are solved exactly, and elsewhere the method of characteristics is used[2]. However, shock-tracking requires special procedures to keep account of all the discontinuities that can arise and, as their number grows, the methods can become rather cumbersome[3].

In contrast, the shock-capturing methods, such as the Lax–Wendroff[4], require no special procedures since the relations across the discontinuities are automatically catered for by the methods themselves. Shock-capturing methods can thus provide reasonably accurate answers to flow problems with a minimum of programming effort.

The Lax–Wendroff method, however, cannot be used on the boundary of a problem so another method is required to approximate the conditions there. Boundary condition approximations have been considered[5,6] for closed and open ends. A hybrid boundary condition approximation, for use with the Lax–Wendroff method, has also been developed which can be used under all types of flow conditions. The method has been tested against exact solutions and has given good accuracy for pipes with open ends. This paper completes this development by showing that the Lax–Wendroff method, in conjunction with this hybrid method of boundary approximation, may be used to calculate the reflected wave from a pipe with a nozzle end. The accuracy of the method is assessed from a comparison with solutions obtained by theoretical methods.

Governing equations

The equations of continuity, momentum and energy for the one-dimensional flow of an ideal gas, with heat transfer and wall-friction, may be written in the form:

$$\frac{\partial V}{\partial t}+\frac{\partial G(V)}{\partial x}=B \tag{1}$$

with

$$V=\begin{bmatrix}\rho\\ m\\ e\end{bmatrix}$$

$$G(V)=\begin{bmatrix}m\\ \rho u^2+p\\ (e+p)u\end{bmatrix}$$

$$B=\begin{bmatrix}0\\ -\rho\phi\\ \rho q\end{bmatrix}$$

and the wall-friction term defined by:

$$\phi=\frac{4f}{D}\frac{u}{2}|u| \tag{2}$$

The particle velocity, u, and the pressure, p, may be obtained from:

$$u=m/\rho \tag{3}$$

and:

$$p=(\gamma-1)(e-\tfrac{1}{2}\rho u^2) \tag{4}$$

The governing equations may also be written in the quasilinear form:

$$\frac{\partial W}{\partial t}+A\frac{\partial W}{\partial x}=C \tag{5}$$

with:

$$W=\begin{pmatrix}\rho\\ u\\ p\end{pmatrix}$$

$$A=\begin{pmatrix}u & \rho & 0\\ 0 & u & 1/\rho\\ 0 & a^2\rho & u\end{pmatrix}$$

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$$C=\begin{pmatrix}0\\-\phi\\(\gamma-1)\rho(q+u\phi)\end{pmatrix}$$

The characteristics and characteristic relations can be obtained from this quasilinear form to give:

$$da\pm\frac{(\gamma-1)}{2}du=\frac{\gamma-1}{2}\alpha dt+\frac{a}{a_a}da_a \qquad (6)$$

with:

$$\alpha=(\gamma-1)\frac{q}{a}\mp\phi\left(1\mp(\gamma-1)\frac{u}{a}\right)$$

on the $C^{(1)}$, $C^{(2)}$ characteristics and:

$$da_a=\frac{1}{2}\frac{a_a}{\rho a^2}\beta dt \qquad (7)$$

with:

$$\beta=(\gamma-1)\rho(q+u\phi)$$

on the C characteristic[4].

Reflected waves from a nozzle end

This section considers the theoretical solution to shock tube problems in which a shock travels down a tube, impinges upon a short nozzle at the end of the tube and undergoes reflection. So that a theoretical solution can be obtained it is assumed that the friction and heat transfer terms are zero in the governing equations.

For a shock wave moving down a tube with a nozzle end the reflected wave may be either a shock wave or a rarefaction wave. Woods[1] discusses graphical methods of solution for the various cases that can arise in pipes with open, closed or nozzle ends. Theoretical solutions are also available and in this paper we consider two of these cases; that of an incident shock producing a reflected rarefaction wave with subsonic flow at the nozzle exit, and that of an incident shock producing a reflected shock wave with sonic flow at the nozzle exit.

Underlying all these solutions is what has been termed[8] the 'quasi-steady assumption'. It is therefore assumed that the length of the nozzle is so short that the flow conditions do not change significantly during the time required for a wave to pass through it. It is thus possible to use the steady state relations across the nozzle as the calculation proceeds. For a converging nozzle these steady state relations choke the flow so that there is always subsonic flow at the nozzle entrance, and either subsonic or sonic flow at the nozzle exit.

In the problems considered here it is assumed that all pipes are of unit length with a short nozzle at the pipe end, $x=1.0$, of area-ratio $\psi=A_e/A$, where A and A_e are the cross-sectional areas of the pipe and nozzle exit respectively. The value of this area-ratio was taken as $\psi=0.5$ in all the examples considered.

A further assumption is that the initial values form a Riemann Initial Value Problem. Initial conditions of the following form are therefore considered:

$$u=u_1 \qquad p=p_1 \qquad \rho=\rho_1 \qquad (8a)$$

for $x<0$, and

$$u=u_0 \qquad p=p_0 \qquad \rho=\rho_0 \qquad (8b)$$

for $x>0$. By choosing these initial conditions to satisfy the Rankine–Hugoniot equations:

$$U(\rho_1-\rho_0)=\rho_1u_1-\rho_0u_0$$
$$U(\rho_1u_1-\rho_0u_0)=\rho_1u_1^2+p_1-\rho_0u_0^2-p_0$$
$$U(e_1-e_0)=u_1(e_1+p_1)-u_0(e_0+p_0) \qquad (9)$$

an exact solution is obtained, consisting of a shock moving with a shock velocity, U, with values u_1, p_1, ρ_1 behind and values u_0, p_0, ρ_0 in front of the shock. This shock may be reflected either as a shock or rarefaction wave, dependent upon the incident shock pressure ratio, z, where $z=(p_1-p_0)/p_0$.

The shock path in (x,t) space can be conveniently represented in a position diagram. The path of a shock, impinging on and reflected from the nozzle as a shock, is shown in Fig 1(a) illustrating the notation. Fig 1(b) shows a shock impinging on a nozzle and reflected as a rarefaction wave.

Reflected rarefaction wave

Assume that the incident shock pressure ratio, z, is small enough in magnitude to produce a reflected rarefaction wave, with subsonic flow at the nozzle exit, once the incident shock has impinged on the nozzle end.

To obtain the solution to the problem, after the shock has impinged on the nozzle, it is necessary to postulate a reflected rarefaction wave behind which the flow has the unknown values: particle velocity, u, and wave velocity, a. The values ahead of the reflected rarefaction wave are u_1 and a_1 and across this wave the Riemann invariant equation:

$$a+\frac{\gamma-1}{2}u=a_1+\frac{\gamma-1}{2}u_1 \qquad (10)$$

Notation

a	Speed of sound
a_a	Speed of sound after isentropic change of state to reference pressure p_{ref}
A,Ae	Cross-sectional area of pipe, nozzle exit
D	Pipe diameter
e	Total energy per unit volume
f	Friction factor
m	Momentum per unit volume
p	Pressure
q	Heat transfer rate per unit mass
t	Time
u	Particle velocity
U	Shock velocity
x	Distance
z	Incident shock pressure ratio
γ	Ratio of specific heats ($\gamma=1.4$)
ρ	Density
ψ	A_e/A

Subscript/superscript

$V_j^n\equiv V(jh,nk)\equiv V(x,t)$

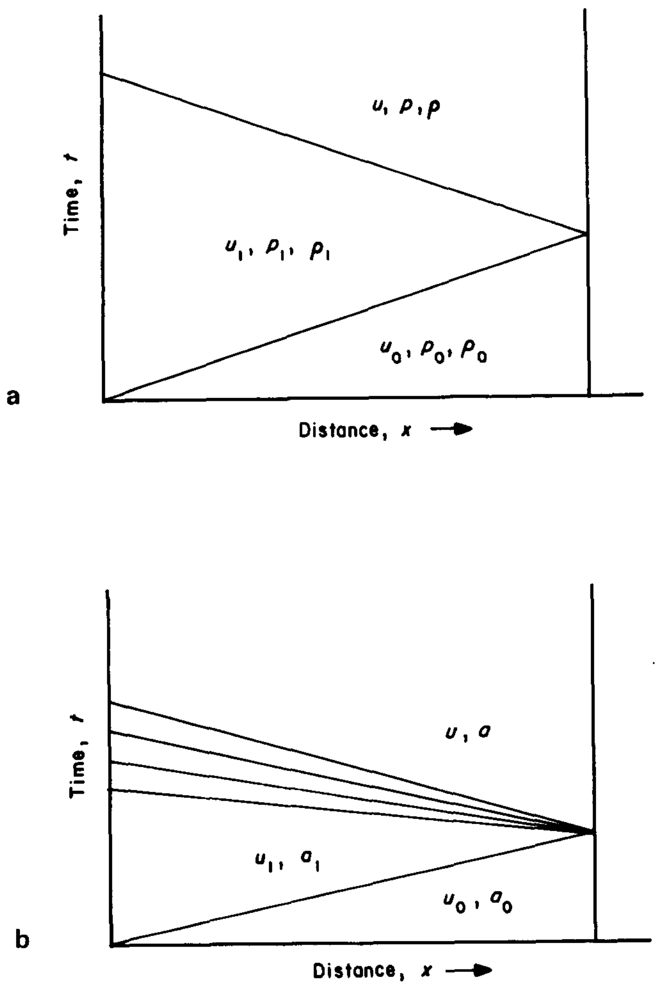

Fig 1 Position diagram (a) reflected shock wave (b) reflected rarefaction wave

must hold. The notation used here is illustrated in the position diagram shown in Fig 1(b).

For subsonic flow at the nozzle exit, the steady-state continuity and energy equations, holding across the nozzle, take the form:

$$\rho u A = \rho_e u_e A_e \tag{11a}$$

$$a^2 + \frac{\gamma-1}{2}u^2 = a_e^2 + \frac{\gamma-1}{2}u_e^2 \tag{11b}$$

where the subscript 'e' indicates values at the nozzle exit. With the further assumption of isentropic flow through the nozzle, Eqs 11(a) and (b) lead to the relation:

$$a^2 + \frac{\gamma-1}{2}u^2 = a_e^2 + \frac{\gamma-1}{2}\left[\left(\frac{a}{a_e}\right)^{2/(\gamma-1)}\frac{u}{\psi}\right]^2 \tag{12}$$

The value of the wave velocity at the nozzle exit, a_e, in this equation is determined by the isentropic relation[4]:

$$a_e = (a_a)_e (p_e/p_{ref})^{(\gamma-1)/(2\gamma)} \tag{13}$$

with the entropy measure variable obtained from $(a_a)_e = (a_a)_1$, and the exit pressure, p_e, assumed known. Using Eq (10) it is now possible to write Eq (12) in terms of the wave velocity, a, and so obtain a non-linear equation which can be solved numerically, eg by the bisection method. This method was used to obtain the solution to the following example.

Incident shock wave with reflected rarefaction wave

The following initial values were assumed:

$u_1 = 0.2988 \qquad p_1 = 1.5 \qquad \rho_1 = 1.867$

for $x < 0$, and:

$u_0 = 0.0 \qquad p_0 = 1.0 \qquad \rho_0 = 1.4$

for $x > 0$.

With these initial values the problem has an exact solution:

$u = 0.2988 \qquad p = 1.5 \qquad \rho = 1.867$

for $x/t < 1.195$ and:

$u = 0.0 \qquad p = 1.0 \qquad \rho = 1.4$

for $x/t > 1.195$ and $0 < t < 0.8368$.

For $t > 0.8368$ the solution consists of two constant state regions separated by a rarefaction fan. In this example the constant values are:

$u = 0.2988 \qquad p = 1.5 \qquad \rho = 1.867$

for $0 \leqslant x \leqslant 1.0 - 0.7622t'$, and:

$u = 0.3135 \qquad p = 1.471 \qquad \rho = 1.841$

for $1.0 - 0.7442t' \leqslant x \leqslant 1.0$, where the time, t', is measured from the moment when the shock arrives at the nozzle end. The region $1.0 - 0.7622t' \leqslant x \leqslant 1.0 - 0.7442t'$ constitutes the rarefaction wave region. These solutions are illustrated in Fig 2 for various values of time. The constant values at the nozzle exit, not shown in the figure, are:

$u = 0.8263 \qquad p = 1.0 \qquad \rho = 1.397$

for $t > 0.8368$.

Reflected shock wave

Assume that the incident shock pressure ratio, z, is large enough to produce a reflected shock, with sonic flow at the nozzle exit, once the incident shock has impinged on the nozzle end.

To obtain the solution to the problem after reflection, it is necessary to postulate a reflected shock wave behind which the flow has the unknown values pressure, p, density, ρ, and particle velocity, u. The values ahead of the reflected shock are p_1, ρ_1, u_1 and across the reflected shock wave the Rankine–Hugoniot equations:

$$\begin{aligned} U(\rho - \rho_1) &= \rho u - \rho_1 u_1 \\ U(\rho u - \rho_1 u_1) &= \rho u^2 + p - \rho_1 u_1^2 - p_1 \\ U(e - e_1) &= u(e + p) - u_1(e_1 + p_1) \end{aligned} \tag{14}$$

must hold, with the shock speed, U, having a negative value, since the shock is now moving to the left against the positive direction of flow. The notation used here is illustrated in the position diagram shown in Fig 1(a).

Following Whitham[7] it is convenient to express these relations in terms of the shock ratio, $z = (p - p_1)/p_1$. The relations across the shock now take the form:

$$\frac{U - u_1}{a_1} = -\left(1 + \frac{\gamma+1}{2\gamma}z\right)^{1/2} \tag{15a}$$

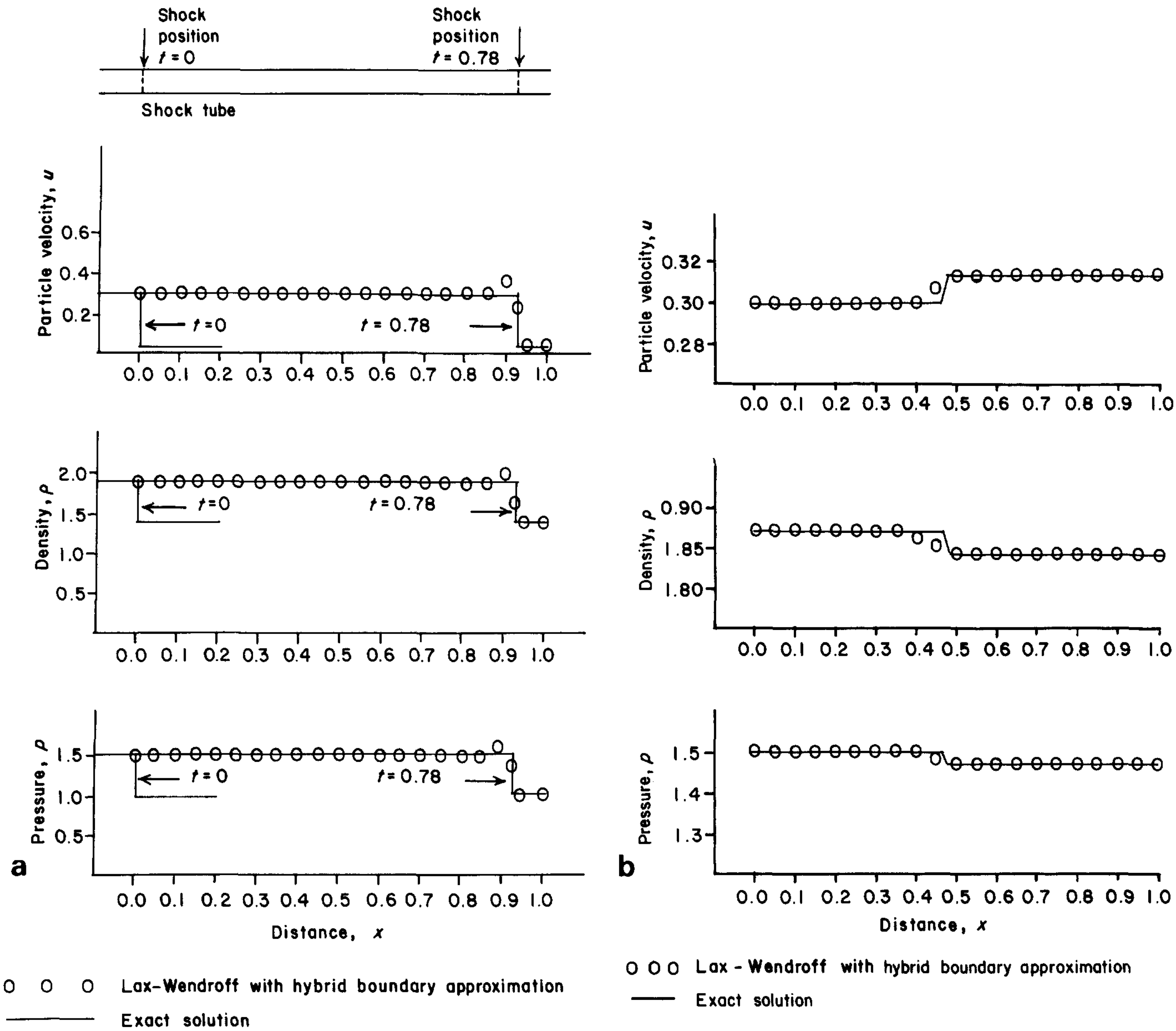

Fig 2 (a) Shock wave, t=*0.0 and* t=*0.78 (b) reflected rarefaction wave,* t=*1.53*

$$\frac{u-u_1}{a_1}=-\frac{z}{\gamma\left(1+\frac{\gamma+1}{2\gamma}z\right)^{1/2}} \tag{15b}$$

$$\frac{\rho}{\rho_1}=\frac{1+\frac{\gamma+1}{2\gamma}z}{1+\frac{\gamma-1}{2\gamma}z} \tag{15c}$$

$$\frac{a}{a_1}=\left(\frac{(1+z)\left(1+\frac{\gamma-1}{2\gamma}z\right)}{1+\frac{\gamma+1}{2\gamma}z}\right)^{1/2} \tag{15d}$$

The problem can now be solved by the determination of the parameter, z, and this can be achieved by examining the flow conditions at the nozzle end. For sonic flow at the nozzle exit, $u_e=a_e$, the continuity and energy equations, holding across the nozzle, take the form:

$$\rho u A=\rho_e u_e A_e \tag{16a}$$

$$a^2+\frac{\gamma-1}{2}u^2=\frac{\gamma+1}{2}a_e^2 \tag{16b}$$

With the further assumption of isentropic flow through the nozzle, valid once the shock wave has been reflected, Eqs (16a) and (16b) lead to the relation:

$$a^2+\frac{\gamma-1}{2}u^2=\frac{\gamma+1}{2}\left(\frac{ua^{2/(\gamma-1)}}{\psi}\right)^{2(\gamma-1)/(\gamma+1)} \tag{17}$$

It is now possible, using Eqs 15(b) and (d), to write Eq (17) in terms of z and so obtain a non-linear equation which can be solved numerically. This method was used to obtain the solution to the following example.

Incident shock with reflected shock wave

The following initial values were assumed:

$u_1=4.0 \qquad p_1=29.0 \qquad \rho_1=7.0$

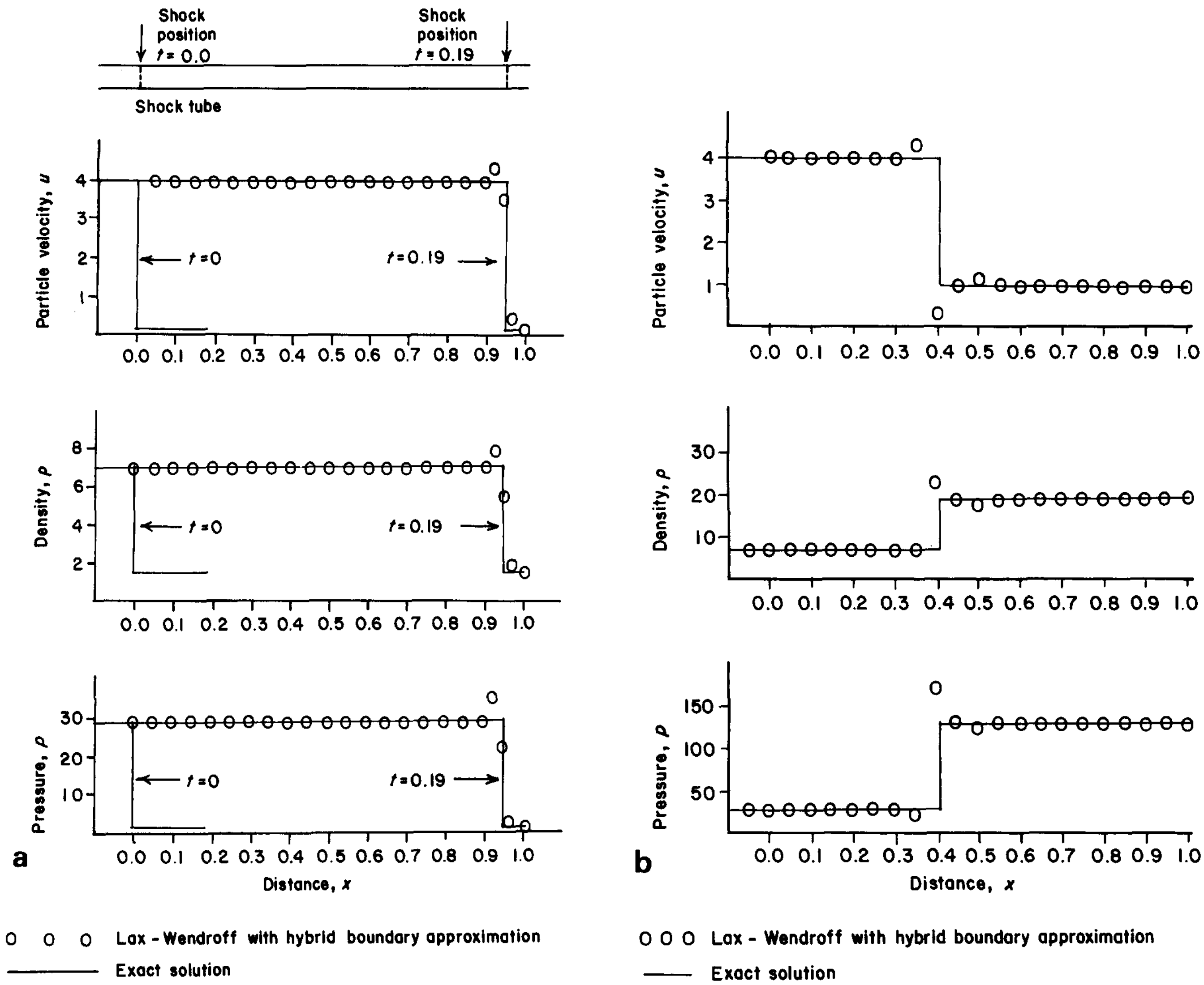

Fig 3 (a) Shock wave, t $=0$ *and* t $=0.19$ *(b) reflected shock wave,* t $=0.9$

for $x<0$, and:

$$u_0=0.0 \qquad p_0=1.0 \qquad \rho_0=1.4$$

for $x>0$.

With these initial values the problem has an exact solution:

$$u=4.0 \qquad p=29.0 \qquad \rho=7.0$$

for $x/t<5.0$ and:

$$u=0.0 \qquad p=1.0 \qquad \rho=1.4$$

for $x/t>5.0$, for all t satisfying $0<t<0.2$.

For $t>0.2$ the solution consists of two constant state regions separated by a shock discontinuity. In this example the constant values are:

$$u=4.0 \qquad p=29.0 \qquad \rho=7.0$$

for $0\leqslant x<1.0-0.8453t'$, and:

$$u=0.9598 \qquad p=132.1 \qquad \rho=18.79$$

for $1.0-0.8453t'<x\leqslant 1.0$, where the time, t', is measured from the moment when the shock is reflected from the nozzle end. These solutions are illustrated in Fig 3 for various values of time. The constant values at the nozzle exit, not shown in the figure, are:

$$u=2.891 \qquad p=74.50 \qquad \rho=12.48$$

for $t>0.2$.

Numerical solution

Two-step Lax–Wendroff method

For a pipe and short nozzle subdivided by the nodal points $j=0,\ldots,J$, as shown in Fig 4, the Lax-Wendroff approximation to Eq (1) gives:

$$V_{j+1/2}^{n+1/2}=\frac{1}{2}(V_{j+1}^{n}+V_{j}^{n})-\frac{\Delta t}{2\Delta x}(G_{j+1}^{n}-G_{j}^{n})+\frac{\Delta t}{4}(B_{j+1}^{n}+B_{j}^{n})$$

$$V_{j}^{n+1}=V_{j}^{n}-\frac{\Delta t}{\Delta x}(G_{j+1/2}^{n+1/2}-G_{j-1/2}^{n+1/2})+\frac{\Delta t}{2}(B_{j+1/2}^{n+1/2}+B_{j-1/2}^{n+1/2}) \tag{18}$$

By applying this approximation to the grid it is possible to calculate V_j^{n+1}, for $j=1,\ldots J-2$. The calculation of V_0^{n+1}, $V_{J-1}^{n+1}$and V_J^{n+1} requires the consideration of the boundary conditions.

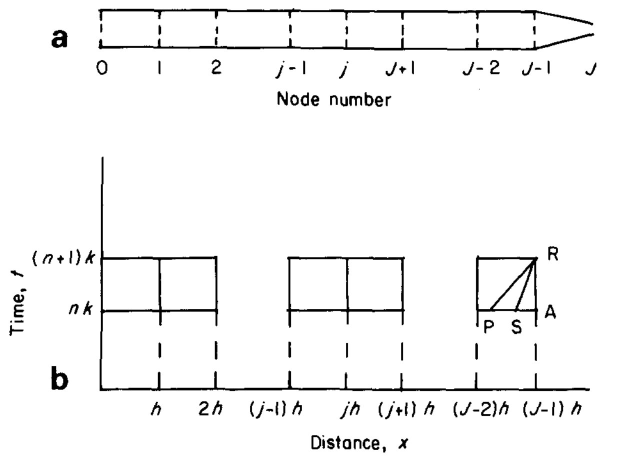

Fig 4 (a) Shock tube with short nozzle (b) computational grid for pipe of length $x=(J-1)h$

Approximation at the outflow boundary

Conditions at the outflow boundary are approximated by the hybrid method, already discussed[6]. In the notation of Fig 4, for the short nozzle considered here, the computation of the flow values at the $J-1$ and J nodes at the time level $t=(n+1)k$ is required. In the hybrid method, the approximation relating the flow between the $J-2$ and $J-1$ nodes is dependent upon whether the flow is subsonic or supersonic at the $J-1$ node. For the converging nozzle considered here, however, the computations showed that there was no possibility of sonic or supersonic flow at the $J-1$ node. This is in agreement with the choking effect of the converging nozzle which limits the flow at the exit end, node J, to sonic, and the flow at node $J-1$ to subsonic values. Since there can only be subsonic flow at node $J-1$, from a consideration of the hybrid method, it now follows that the flow between nodes $J-2$ and $J-1$ is governed solely by the characteristic equations.

In the numerical approximation many of the equations already used still hold, thus, for brevity, the following redefinitions are introduced:

$$a_J^{n+1}=a_e \qquad u_J^{n+1}=u_e \qquad (a_a)_J^{n+1}=(a_a)_e$$

$$a_{J-1}^{n+1}=a, \qquad u_{J-1}^{n+1}=u \qquad (a_a)_{J-1}^{n+1}=a_a$$

$$a_P=a_1 \qquad u_P=u_1 \qquad (a_a)_S=(a_a)_1$$

using the notation of Fig 4. These redefinitions apply to Eqs (11)–(13), (16) and (17). At the nozzle exit the flow may be either subsonic or sonic and these cases are now considered separately.

For subsonic flow at the nozzle exit it is once again possible to write Eq (12) as a non-linear equation in terms of the wave velocity, a_{J-1}^{n+1}, by using the numerical approximation to Eq (6), along the $C^{(1)}$ characteristic:

$$a_R+\frac{\gamma-1}{2}u_R=a_p+\frac{\gamma-1}{2}u_p+\frac{\gamma-1}{2}\alpha_A k+\frac{a_A}{(a_a)_A}[(a_a)_R-(a_a)_p] \tag{19}$$

to eliminate $u_{J-1}^{n+1}(=u_R)$. The value of the entropy measure variable, $(a_a)_R$, may be obtained from the numerical approximation to Eq (7), along the C characteristic:

$$(a_a)_R=(a_a)_S+\frac{1}{2}\frac{(a_a)_A}{\rho a_A^2}\beta_A k \tag{20}$$

Once the values at $[(J-1)h, (n+1)k]$ have been obtained Eqs (11a) and (11b) may be used to obtain values at the nozzle exit, $[Jh, (n+1)k]$.

For sonic flow at the nozzle exit, Eq (17) may be written as a non-linear equation in terms of the wave velocity, a_{J-1}^{n+1}, in a similar fashion. Once the values at $[(J-1)h, (n+1)k]$ have been obtained Eqs (16a) and (16b), along with the isentropic assumption, may be used to obtain the values at $[Jh, (n+1)k]$.

These subsonic and sonic approximations are used in the following manner. In the numerical solution it is assumed initially that the flow is subsonic at the nozzle exit and the subsonic approximations are used. If the computed flow at the nozzle exit gives a subsonic solution the calculation is terminated, but if supersonic flow values are obtained the solutions are recalculated using the sonic approximations.

Computational notes

Computations were performed to check the accuracy of the numerical method in comparison with that obtained from the theoretical solutions. Hence all the computations were performed with the assumption of zero friction, ($\phi=0$), zero heat transfer ($q=0$), and the nozzle was taken to have an area ratio, $\psi=0.5$. Solutions were obtained for various mesh-lengths, Δx, with the time increment, Δt, obtained from $\max(|u|+a)\Delta t/\Delta x=0.9$ in order to satisfy the Courant-Friedrichs-Lewy condition[4]. The results of these computations are shown in Figs 2 and 3.

As the illustrated solutions to these examples show, the Lax-Wendroff method with the hybrid boundary approximation can achieve a good representation of the theoretical solutions. The values at the nozzle exit are not shown in the figures. For the example with the rarefaction wave, the values obtained by the Lax-Wendroff method were:

$$u=0.8260 \qquad p=1.0 \qquad \rho=1.397$$

for $t>0.8368$, showing agreement to three significant figures with the theoretical solution. For the example with the reflected shock wave, the values obtained were:

$$u=2.887 \qquad p=74.18 \qquad \rho=12.46$$

for $t>0.2$, showing agreement to two significant figures with the theoretical solution.

Conclusion

This paper has been concerned with the computation of gas flow in pipes with a short nozzle end. The computation of solutions using a shock-capturing method, the Lax-Wendroff method with a hybrid method of approximation at the boundary, has been considered. The accuracy of this method has been assessed by comparison with theoretical solutions, involving both shock and rarefaction reflections, through a choice of suitable parameters. These theoretical solutions may be used as a benchmark for the comparison of numerical methods of solution. Solutions to the test problems using the Lax-Wendroff method with the hybrid boundary condition approximation have shown that good accuracy can be achieved.

Finally, the method developed here could be applied to autoclave systems under disc rupture, and it is intended to report on some theoretical/practical comparisons shortly.

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BOOK REVIEW

Review of Mass Flow Measurements – 1984

Eds T. R. Hedrick and R. M. Reimer

The papers compiled in this document reflect the wide range of concerns of flow meter manufacturers and users alike that increasingly prevail in many industrial sectors in 1985. The current scarcity of critical fluid resources and the rising prices of valuable fluid products are generating enhanced interest in improved fluid measurements. This is true not only for the accuracy requirements in custody transfer between buyer and seller of fluid but also in the precision levels now needed to satisfactorily monitor and control fluid flow for optimal productivity in the process industries, particularly the chemical and petrochemical industries.

Mass flowrate is one of the fundamental physical variables in fluids engineering. Its measurement is an essential part of the development of many new process systems. The papers collected in this document represent a compilation of the flowmetering efforts underway today to improve fluid measurement. These papers describe:

(1) new techniques or concepts in instrumentation;
(2) experimental or test results to establish accuracy and/or precision of flow measurement techniques;
(3) new methods or analytical models used to interpret instrument response characteristics.

The papers are divided into four sections—two of these are devoted to differential pressure-type fluid metering topics, one to multiphase flow measurement, and one to electrical type fluid meters.

Of the two differential pressure sections, one deals entirely with orifice meters and reflects the increased concerns prevailing today in the large-volume custody transfer of gas using these devices. The large calibration-testing programme underway both in the USA and in Europe to improve the fundamental data base for orifice meters indicate both the widespread concern and level of interests in improving matters. Correspondingly, a number of studies have been made to understand salient features of orifice flow. Several of these involving geometrical or fluid dynamic effects are presented in the orifice metering section of this document. The remaining papers address alternative techniques such as vortex shedding, venturis, nozzles, etc.

The multiphase flow measurement section contains a number of papers dealing with various aspects of the multitude of variables and conditions that are significant in this area of fluid (and solids) measurement. Paper topics range from the assessment of techniques determining the mass flowrate of gas-liquid mixtures through the analysis of capacitive techniques for void fraction to the presentation of test results for multi-component systems for measuring mass flowrate in transient two-phase flows.

The electrical-type fluid metering section contains four papers which reflect efforts to establish new or improve existing measurements using electrical effects. The range of topical areas addressed includes droplet flux in mist flows, to thermal probe arrays for the duct flow of air, to velocity measurement via charging techniques in low conductivity fluids.

In addition to concluding that this is a broad ranging and interesting compendium of fluid measurement results, the Coordinating Group on Fluid Measurements and Fluid Meters Committee, of the Fluids Engineering Division of ASME is to be commended for producing such a worthwhile document.

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